

Chapter 5 – Polynomial Functions

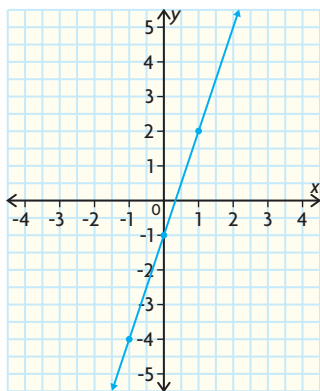
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1. b), c), d)

2.

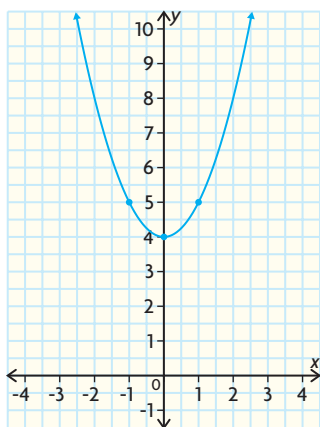
	Domain	Range	x-Intercepts	Number of Turning Points	y-Intercept	End Behaviour
b)	$\{x \in \mathbb{R}\}$	$\{y \mid y \geq -2.5, y \in \mathbb{R}\}$	-5, -1	1	2	QII to QI
c)	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$	-2, -1, 1	2	-5	QIII to QI
d)	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$	0.5	0	2	QII to QIV

3. a)



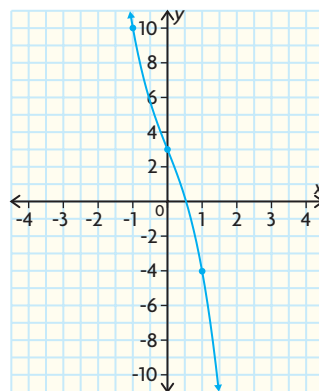
End Behaviour	y-Intercept	Number of Turning Points	Number of x-Intercepts	Domain	Range
QIII to QI	-1	0	1	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$

b)



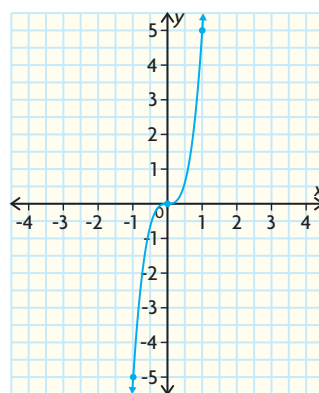
End Behaviour	y-Intercept	Number of Turning Points	Number of x-Intercepts	Domain	Range
QII to QI	4	1	0	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R} \mid y \geq 4\}$

c)



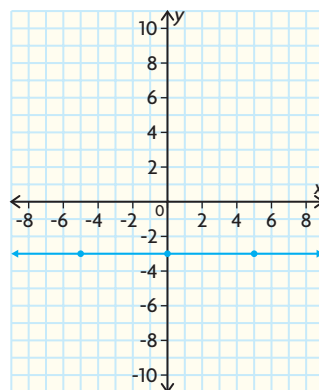
End Behaviour	y-Intercept	Number of Turning Points	Number of x-Intercepts	Domain	Range
QII to QIV	3	0	1	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$

d)

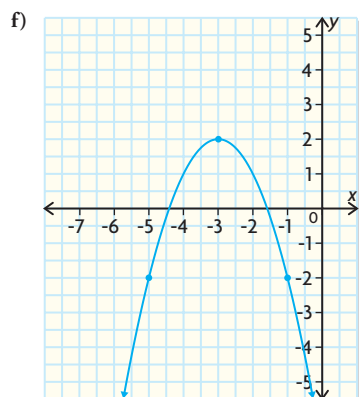


End Behaviour	y-Intercept	Number of Turning Points	Number of x-Intercepts	Domain	Range
QIII to QI	0	0	1	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$

e)

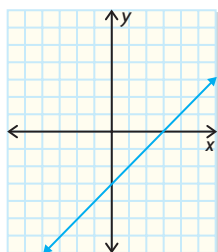


End Behaviour	y-Intercept	Number of Turning Points	Number of x-Intercepts	Domain	Range
QIII to QIV	-3	0	0	$\{x \in \mathbb{R}\}$	$\{y = -3, y \in \mathbb{R}\}$

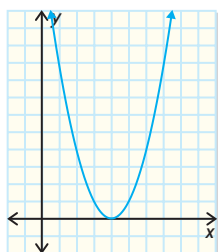
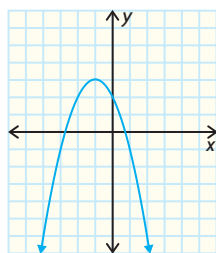
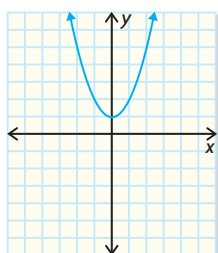


End Behaviour	y-Intercept	Number of Turning Points	Number of x-Intercepts	Domain	Range
QIII to QIV	-7	1	2	$\{x \in \mathbb{R}\}$	$\{y \mid y \leq 2, y \in \mathbb{R}\}$

4. a) 1 x-intercept



b) 0, 1, or 2 x-intercepts



c) 1, 2, or 3 x-intercepts



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1.

	Degree	Leading Coefficient	Constant
a)	2	6	-2
b)	1	$-\frac{2}{3}$	10
c)	3	-1	6
d)	3	4	-10

2.

- minimum x-intercepts: 0, maximum x-intercepts: 2
 - end behaviour: QII to QI, domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R} \mid y \geq \text{minimum}\}$
 - minimum turning points: 1, maximum turning points: 1
- minimum x-intercepts: 1, maximum x-intercepts: 1
 - end behaviour: QII to QIV, domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}\}$
 - minimum turning points: 0, maximum turning points: 0
- minimum x-intercepts: 1, maximum x-intercepts: 3
 - end behaviour: QII to QIV, domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}\}$
 - minimum turning points: 0, maximum turning points: 2
- minimum x-intercepts: 1, maximum x-intercepts: 3
 - end behaviour: QIII to QI, domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}\}$
 - minimum turning points: 0, maximum turning points: 2

3.

	Degree	Sign of Leading Coefficient	Constant Term
a)	2	-	2
b)	1	+	2
c)	3	+	-1
d)	3	-	6

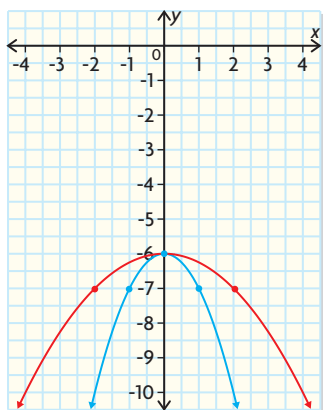
4.

- e.g., $y = 5$
- e.g., $y = 3x + 5$
- e.g., $y = 4x^2 + 3x + 5$
- e.g., $y = -5x^3 + x^2 - x + 5$

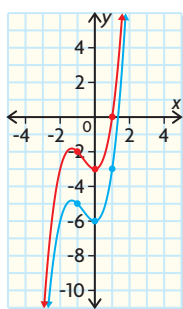
5. a) QIII to QI c) QII to QI e) QII to QIV
 b) QIII to QIV d) QII to QIV f) QIII to QI
 6. a) v) b) i) c) ii) d) vi) e) iii) f) iv)
 7.

End Behaviour	Number of Turning Points	Number of x -Intercepts	y -Intercepts	Domain	Range
a) QII to QIV	0	1	5	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$
b) QII to QI	1	0, 1, or 2	-6	$\{x \in \mathbb{R}\}$	$\{y \mid y \geq \text{minimum}, y \in \mathbb{R}\}$
c) QIII to QI	0 or 2	1, 2, or 3	-1	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$
d) QII to QIV	0 or 2	1, 2, or 3	0	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$

8. a) e.g., $y = -x^2 + 2$
 b) e.g., $y = x^3 + 2x^2 - x - 2$
 c) e.g., $y = x - 3$
 d) e.g., $y = x^3 + 2x^2 - x + 5$
 e) e.g., $y = -x^2 + 2$
 9. e.g., Lukas made an error when he stated, "There is no constant term, so there is no y -intercept." Since there is no constant term, there is a y -intercept at $y = 0$.
 10. a) e.g.,



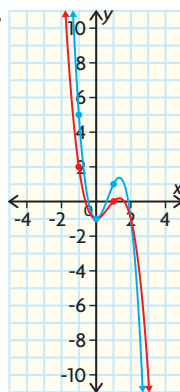
b) e.g.,



c) e.g.,



d) e.g.,

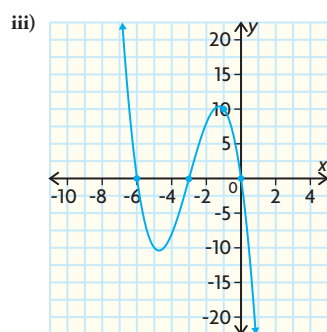
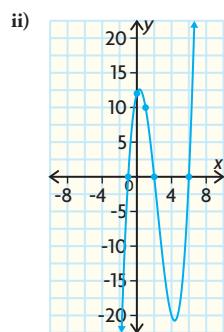
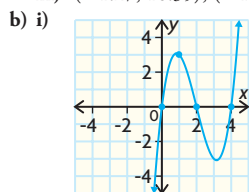


11. e.g., A cubic function may have zero or two turning points. If there are no turning points, the function has only one x -intercept. If there are two turning points, the function may have one, two or three x -intercepts, depending on the values of a , b , c , and d in the function $y = ax^3 + bx^2 + cx + d$ (see figures shown).



12. e.g., Cubic functions may have turning points, but they do not have any maximums or minimums.
 Quadratic functions have one turning point. The point at which they turn (the vertex) defines the function's maximum or minimum.
 13. a) i) b) iii) c) iv) d) ii)
 14. a) The degree is 3, so it is a cubic function. The leading coefficient is positive, so the function is increasing from left to right. The graph extends from quadrant III to quadrant I.
 It has a y -intercept of 25.720 and may have 1, 2, or 3 x -intercepts.
 b) the price of gas in 1979
 15. a) 4.2 m; It is very close.
 b) extends from QIII to QI
 c) e.g., the tides change every day, as shown in the table for Jan. 10
 16. e.g., Ask for the degree of the function to determine the end behaviour. Ask for the value of the leading coefficient to determine which quadrants the end behaviour extends to and whether the function is increasing or decreasing from left to right. Ask for the number of x -intercepts to determine how many times the function crosses the x -axis. These questions will enable you to draw a rough sketch of the graph and determine a plausible equation.

17. a) i) (0.85, 3.08), (3.15, -3.08)
 ii) (0.31, 12.60), (4.36, -20.75)
 iii) (-1.27, 10.39), (-4.73, -10.39)



- c) e.g., The x -coordinate occurs approximately at the midpoint between the x -intercepts. Once this midpoint is found, substitute this x -value into the equation to determine the y -coordinate. e.g., iii) the x -intercepts are -6 , -3 , and 0 . The midpoint between -6 and -3 is -4.5 ; find the y -value when x is -4.5 . The midpoint between -3 and 0 is -1.5 ; find the y -value when x is -1.5 .

18. a) e.g., $P(0) = 0$, $P(0.5) = 0.375$, $P(1) = 0$; The function seems reasonable.
 b) $D: \{x \mid 0 \leq x \leq 1, x \in \mathbb{R}\}$; $R: \left\{y \mid y \leq \frac{4}{9}, y \in \mathbb{R}\right\}$
 c) $\left(\frac{2}{3}, \frac{4}{9}\right)$; e.g., At this point, the probability of sinking two out of three free throws begins to decrease.
 d) $x = 0, 1$; e.g., The probability of sinking two out of three free throws is zero if the probability of sinking one throw is 0. The probability of sinking exactly two out of three free throws is 0 if the probability of sinking one throw is 1 (that is, you'd be more likely to sink 3 out of 3).

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1. a), c), d)

Degree	Domain	Range	Constant
a) 2	$\{x \in \mathbb{R}\}$	$\{y \mid y \leq 6, y \in \mathbb{R}\}$	5
c) 0	$\{x \in \mathbb{R}\}$	$\{y \mid y = 4, y \in \mathbb{R}\}$	4
d) 3	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$	-2

- b) i) d) ii) c) iii) a)

3. a) QII to QIV b) QIII to QI c) QIII to QI d) QIII to QIV

Degree	End Behaviour	x -Intercepts	y -Intercept	Domain	Range	Number of Turning Points
i) 3	QII to QIV	5	4	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$	2
ii) 2	QIII to QI	-1.5, 2	-6	$\{x \in \mathbb{R}\}$	$\{y \mid y \geq -6.25, y \in \mathbb{R}\}$	1
iii) 1	QII to QIV	6.5	2	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$	0
iv) 3	QIII to QI	-1, 3	9	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$	2

- b) i) leading coefficient: $-$, constant: 4
 ii) leading coefficient: $+$, constant: -6
 iii) leading coefficient: $-$, constant: 2
 iv) leading coefficient: $+$, constant: 9

- 5.

End Behaviour	Number of Turning Points	Number of x -Intercepts	y -Intercept	Domain	Range
a) QIII to QIV	1	2	6	$\{x \in \mathbb{R}\}$	$\{y \mid y \leq \text{maximum}, y \in \mathbb{R}\}$
b) QIII to QI	0 or 2	1, 2, or 3	6	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$
c) QII to QI	1	0, 1, or 2	-1	$\{x \in \mathbb{R}\}$	$\{y \mid y \geq \text{minimum}, y \in \mathbb{R}\}$
d) QIII to QI	0 or 2	1, 2, or 3	0	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$

6. a) e.g., $y = (x + 1.5874)^3 - x - 4$
 b) e.g., $y = (x - 3)(x + 4)$
 c) e.g., $y = -x - 3$
 d) e.g., $y = 1.5(x - 2)^2$
 e) e.g., $y = 1.5(x - 4)^2 - 6$

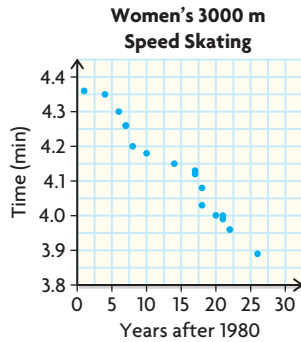
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1. a) e.g., slope: -1 , y -intercept: 7.5
 b) e.g., slope: 0.1 , y -intercept: 3
 2. a) e.g., $y = -x + 7.5$
 b) e.g., $y = 0.1x + 3$

Independent	Dependent
a) speed	distance
b) size of family	number of cellphones
c) time of day	number of people
d) time of year	hours of daylight

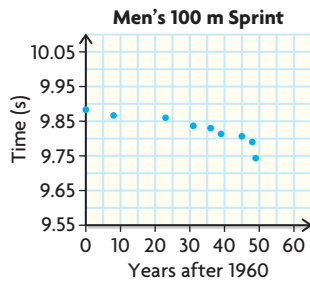
4. a) e.g., The line of best fit has a negative slope and has about the same number of points above and below it. The points are very close to the line of best fit, so it should give a strong representation of the data.
 b) e.g., about 70; interpolation, because the point is between known values
 c) e.g., about 52; interpolation, because the point is between known values
 d) e.g., about 110; extrapolation, because the point is outside known values

5. a)



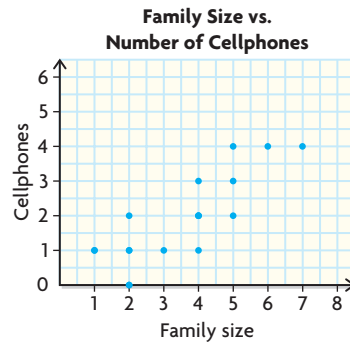
- b) As the years increase, the world-record times decrease.
 c) $y = -0.019x + 4.396$ (where y is time in seconds and x is the year); The slope represents the rate at which the world-record time decreases each year, in minutes; the y -intercept represents the world-record time in 1980 (year 0).
 d) e.g., 3:55.26 min
 e) 3:53.34 min; My estimate was very close.

6. a)



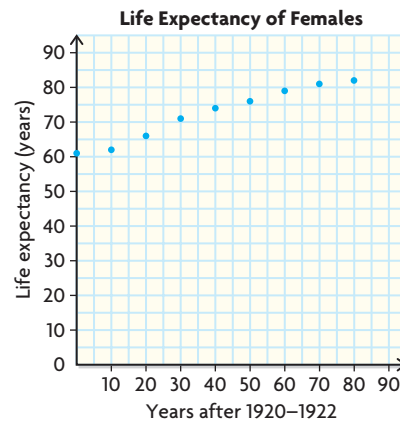
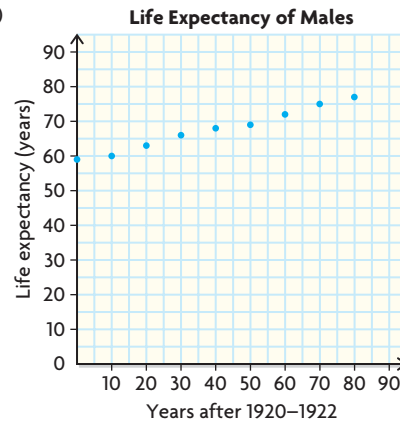
- b) As the years increase, the world-record times decrease.
 c) $y = -0.006x + 10.0$; The slope represents the rate at which the world-record time in seconds decreases each year; the y -intercept represents the world-record time in 1960 (year 0).
 d) e.g., 9.75 s
 e) 9.74 s; My estimate was very close.
 7. a) e.g., As latitude increases (i.e., you go further north), the mean temperatures would decrease. My plot verifies that my prediction is correct.
 b) $y = -0.637x + 49.73$
 c) 16.6 °C
 d) 49.8°N

8. a) As family size increases, the number of cellphones increases.
 b)



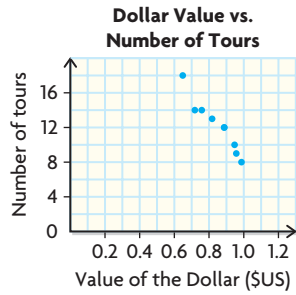
$$y = 0.614x + 0.260$$

- c) e.g., 2.1 cellphones; I would expect a family of 3 to have 2 cellphones.
 9. 2 events
 10. 109 bikes
 11. a)



- b) males: $y = 0.23x + 58.46$; females: $y = 0.286x + 60.97$
 c) males: 79 years; females: 87 years

12.



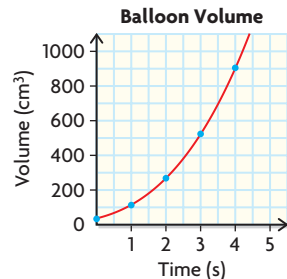
$$y = -24.88x + 33.315$$

e.g., Using extrapolation, the expected number of tours is 8. There is a strong linear relationship between the value of the dollar and the number of tours.

13. a) e.g., Create a scatter plot of the data and perform a linear regression. Insert the known independent value for x and solve for y . If the regression yields $y = 0.5x + 2$, and you want to estimate the value of the dependent variable when the independent variable is 4, input 4 for x and solve for y ; this gives us a y -value of 4.
b) e.g., From the linear regression, substitute the known value of the dependent variable for y , and solve for x (the independent variable).
14. a) $y = 2.445x - 23.119$
b) -10.9 board feet; e.g., This answer does not make sense, because area cannot be negative. Extrapolating does not always make sense, because the trend does not always continue outside the given data range.
c) e.g., Based on the scatter plot, the data does not seem to be linear.
d) e.g., Exponential regression may be better because the data fit an exponential function.

Lesson 5.4, page 313

1. a) e.g., The number of births increases from 1947 to about 1960; after that it decreases at about the same rate until 1971.
b) about 1960
c) about 425 000
d) between 1953 and 1965
2. a) 149 bpm b) about 244 W
3. a) The ball rises to a maximum height at about 3 s and then its height decreases.
b) $y = -10.07x^2 + 51.409x + 11$
c) i) 11 m ii) 76.6 m iii) 38.4 m
d) about 5.3 s
4. a), b)



$$y = 4.189x^3 + 25.131x^2 + 50.268x + 33.51$$

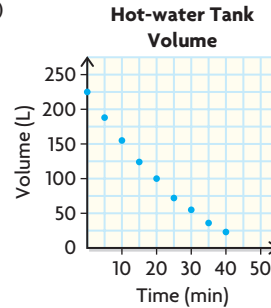
e.g., As time increases, volume increases at a faster rate.
c) 8181.31 cm³

5. a) 3.9 °C b) 1006.55 cm³

6. 1.4 m

7. a) e.g., 2 years after 1977, fertility rate is about 3.4 per 1000 females; 7 years after 1977, fertility rate is about 3.4318 per 1000 females
b) e.g., The value when $x = 2$ is close; the value when $x = 7$ is not as close.

8. a)



e.g., As time increases, the volume decreases.

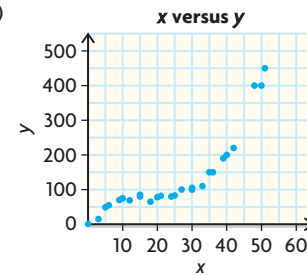
b) $V = 0.0649t^2 - 7.65t + 224.88$

c) 17.2 s

d) 56.2 s

9. a) $y = 0.004...x^3 - 0.294...x^2 + 4.548...x + 74.659...$; 65.332...
b) Never. The minimum of 65.3 per 100 000 is reached between 2009 and 2010.

10. a)



b) $y = 0.01x^3 - 0.557x^2 + 11.08x + 1.141$

c) 86.3; the result from the regression equation is off by 3.3.

11. a) e.g., Linear: $y = 115.83x + 7000$ for $0 \leq x \leq 9$, Quadratic: $y = -9.7832x^2 + 311.4x + 6032.2$ for $10 \leq x \leq 22$, Linear: $y = -115.92x + 10\,689$ for $23 \leq x \leq 30$
b) e.g., about 15 s

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1. a) i) e.g., $y = -x + 3$
ii) e.g., $y = 14x - 6$
b) i) e.g., $y = -x^3 + 3$
ii) e.g., $y = x^3 - 6$
2.

End Behaviour	Number of Turning Points	Number of x-Intercepts	y-Intercept	Domain	Range
a) QIII to QIV	1	2	-1	$\{x \in \mathbb{R}\}$	$\{y \mid y \leq 3, y \in \mathbb{R}\}$
b) QIII to QI	0	1	1.5	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$
c) QIII to QI	2	3	-42	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$
d) QII to QIV	2	3	2	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$

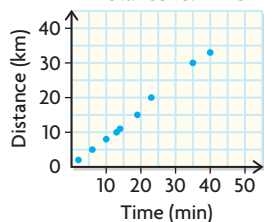
3. a) e.g., $y = -x^3 + 2x - 6$
b) e.g., $y = -x^2 + 2x + 8$

4.

	Number of Turning Points	End Behaviour	Number of x-Intercepts	y-Intercept	Domain	Range
a)	1	QII to QI	0 or 2	12	$\{x \in \mathbb{R}\}$	$\{y \mid y \geq 4, y \in \mathbb{R}\}$
b)	0	QII to QIV	1, 2, or 3	-1	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$

5. e.g., The line of best fit would have a negative slope of about -1 and about the same number of points above and below the line. The y -intercept would be about 125.

6. a) **Distance vs. Time**



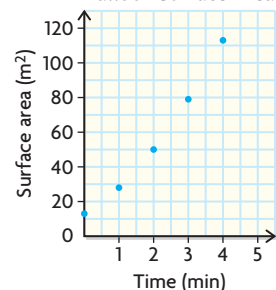
b) $y = 0.844x - 0.309$

c) i) about 29 min

ii) about 38 km

iii) about 50.6 km/h

7. a) **Balloon Surface Area**



b) $A = 3.214t^2 + 12.24t + 12.83$

c) e.g., interpolate: at 1.5 min, surface area would be about 38.4 m²;
extrapolate: at 5 min, surface area would be about 154.4 m²

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1. a), d), and e)

2.

	Domain	Range	Number of x-Intercepts	Number of Turning Points	y-Intercept
b)	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$	1, 2, or 3	0 or 2	-4
c)	$\{x \in \mathbb{R}\}$	$\{y \mid y \leq 0, y \in \mathbb{R}\}$	0, 1, or 2	1	-32

3.

	End Behaviour	Domain	Range	y-Intercept
a)	QIII to QIV	$\{x \in \mathbb{R}\}$	$\{y \mid y \leq \text{maximum}, y \in \mathbb{R}\}$	-1
b)	QIII to QI	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$	5
c)	QII to QI	$\{x \in \mathbb{R}\}$	$\{y \mid y \geq -9, y \in \mathbb{R}\}$	0
d)	QIII to QI	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$	5

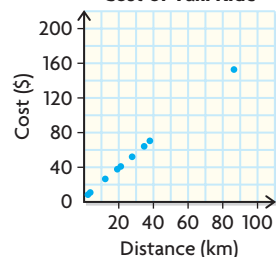
4.

	End Behaviour	Domain	Range	Maximum Number of x-Intercepts	y-Intercept	Number of Turning Points
a)	QIII to QIV	$\{x \in \mathbb{R}\}$	$\{y \mid y \leq -4, y \in \mathbb{R}\}$	0	-4.5	1
b)	QIII to QI	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$	3	0	2

5. a) e.g., slope: $\frac{1}{5}$, y -intercept: 0.5

- b) e.g., slope: -2 , y -intercept: 6

6. a) **Cost of Taxi Ride**



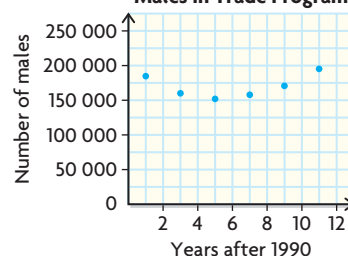
As the distance increases, the cost increases in a linear relationship.

b) $y = 1.711x + 4.966$

c) i) \$90.52 ii) \$4.97

7. a) 22 m b) 1.06 s

8. a) **Males in Trade Program**



b) $y = 1471x^2 - 16\,364x + 198\,122$

Years After 1990	Number of Males
2	171 278
4	156 202
6	152 894
8	161 354
10	181 582
12	213 578

9. a) $y = 7.0624x^3 - 77.36x^2 + 1070.0x + 7208.2$
b) 2002

10. a) $y = -5x^2 + 20$
b) about 1.4 s

Cumulative Review, Chapters 4–5, page 327

1. a) i) e.g., $\frac{2}{a}$ ii) e.g., $\frac{12x}{2(x-3)}$
b) i) $a \neq 0$ ii) $x \neq 3$
c) i) e.g., $a = 1$: $\frac{18a^2}{9a^3} = \frac{18(1)^2}{9(1)^3} = \frac{18}{9} = 2$, $\frac{2}{a} = \frac{2}{1} = 2$
ii) e.g., $x = 1$: $\frac{6x}{x-3} = \frac{6(1)}{1-3} = \frac{6}{-2} = -3$,
 $\frac{12x}{2(x-3)} = \frac{12(1)}{2(1-3)} = \frac{12}{-4} = -3$

2. a) i) $y \neq \frac{1}{2}$ ii) $x \neq 2$
b) i) e.g., $\frac{2y^2 + y}{2y^2 - y}$, $\frac{4y + 2}{4y - 2}$ ii) e.g., $\frac{x - 2}{2x - 4}$, $\frac{1}{2}$

3. Yes. e.g., Multiply or divide both the numerator and denominator by the same value so that you are multiplying or dividing by 1.

For example, multiply $\frac{x+2}{3}$ by $\frac{2}{2}$ to get the equivalent expression $\frac{2x+4}{6}$.

4. a) $x \neq 0$, $-\frac{5}{x^2}$ b) $a \neq 2, -2$, $\frac{1}{2(a-2)}$

5. e.g., It is similar because you can factor and divide by like factors in both types. It is different because a rational expression has variables and a rational number does not.

6. a) $\frac{9x}{4}$, $x \neq 0$ c) $\frac{2}{3y^3}$, $y \neq 0$
b) $9a^5$, $a \neq 0$ d) $2m$, $m \neq 0$

7. a) 3 , $a \neq -2, 2$ b) $\frac{-3(3y-2)}{2(3y+2)}$, $y \neq 3, -\frac{2}{3}$

8. a) $\frac{4x+3}{6x^3}$ b) $\frac{3y^2 - 26y}{y^2 - y - 12}$

9. a) $x \neq 0$; $x = 3$ b) $x \neq -2, -3$; $x = -1, -\frac{27}{13}$

10. Eric: 51 min 36 s, Sid: 71 min 36 s

End Behaviour	Domain	Range	x-Intercepts	y-Intercept
a) QIII to QI	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$	$-3, \pm 2$	-12
b) QII to QIV	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$	$\pm 3, 1$	-9

Degree	Sign of Leading Coefficient	Constant	Number of Turning Points
a) 3	+	12	2
b) 3	−	−9	2

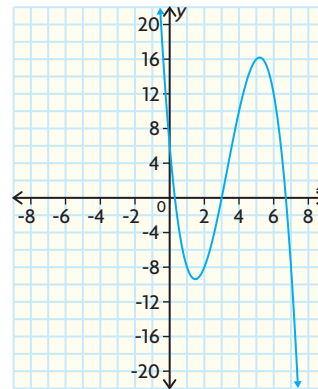
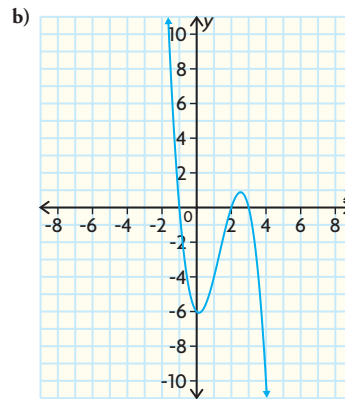
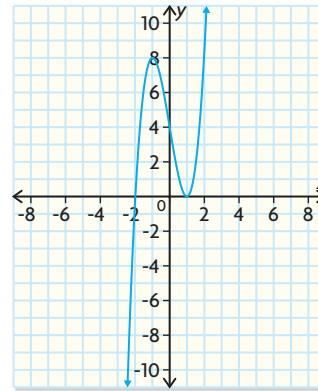
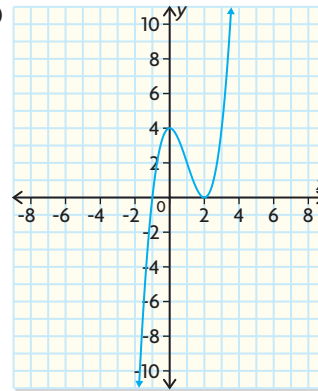
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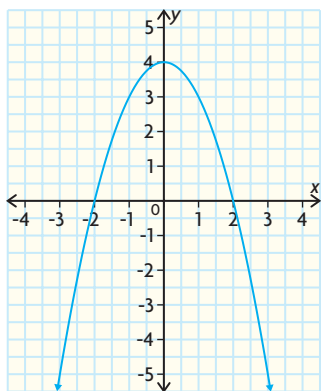
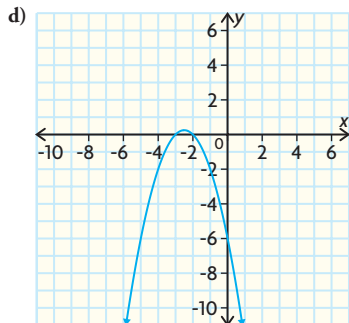
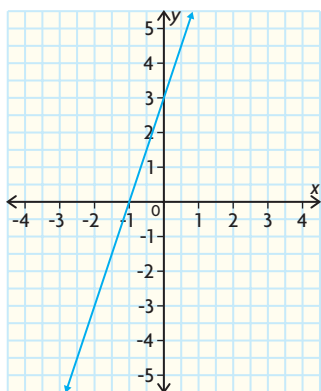
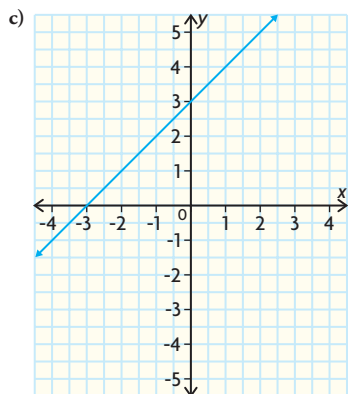
End Behaviour	Number of Turning Points	Number of x-Intercepts	y-Intercept	Domain	Range
a) QIII to QI	0	1	-2	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$
b) QIII to QIV	1	0, 1 or 2	-8	$\{x \in \mathbb{R}\}$	$\{y \mid y \leq \text{maximum}, y \in \mathbb{R}\}$
c) QIII to QI	0 or 2	1, 2 or 3	-2	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$
d) QII to QIV	0 or 2	1, 2 or 3	0	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$

$\{y \mid y \leq \text{maximum}, y \in \mathbb{R}\}$

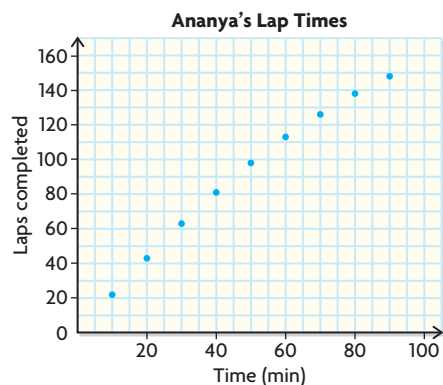
NEL

14. a)





15. a)

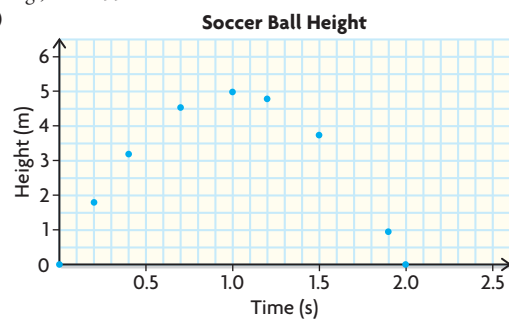


$$y = 1.5783x + 13.528$$

b) e.g., about 130 laps

c) e.g., about 55 min

16. a)

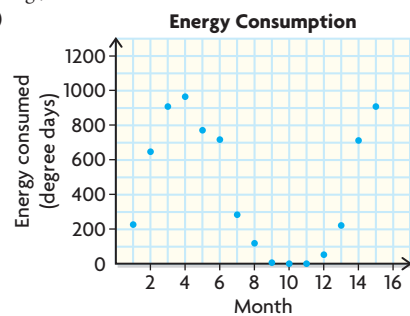


$$y = -5x^2 + 10x$$

b) e.g., 0.35 s, 1.65 s

c) e.g., 3.75 m

17. a)



$$y = 4.8087x^3 - 106.41x^2 + 597.47x - 163.8$$

b) e.g., middle of eighth month to end of twelfth month

c) e.g., middle of seventh month: 360.3 degree days, middle of eighth month: 179.7 degree days